Idiosyncratic choice bias in a decision-making task

We consider a decision-making network that two populations of neurons, denoted by 1 and 2, representing two choices. Each population consists of $\frac{N}{2}$ independent Poisson neurons and we assume that $N \gg 1$. The firing rates of the neurons are independently and identically drawn from a distribution such that the $\mathrm{E}[\nu_i^\alpha] = \nu$ and $\mathrm{Var}[\nu_i^\alpha] = \sigma^2$.

In each trial the cumulative number of spikes, n^1 and n^2, emitted by populations 1 and 2 up to time T is counted. If $n^1 > n^2$ then action 1 is chosen. Otherwise, if $n^1 < n^2$ then action 2 is chosen.

(1) Simulate decision making in networks with $N = 1000$ neurons, when $\nu = 1$ and $\sigma^2 = 1$:
   (a) For $T = 1$, randomly draw one network and estimate the probability that action 1 is chosen by simulating the decision process 100 times. Is network biased towards one of the actions?

   (b) Repeat the simulation for 100 different networks and plot the distribution of probabilities of choosing action 1 across the networks.

   (c) How does the shape of the distribution depend on N? Repeat the simulation for different values of N.

   (d) How does the shape of the distribution depend on T? Try different values of T.

(2) Analytical analysis:
   (a) For a specific realization of firing rates of the neurons of the two populations, write an expression for the probability P^1 that action 1 will be chosen as a function of the population firing rates $\nu^\alpha = \sum_{i=1}^{N} \nu_i^\alpha$ and T. The answer should depend on the firing rates through $\Delta\nu \equiv \nu^1 - \nu^2$ and $S\nu \equiv \nu^1 + \nu^2$. Compare the results to the simulations of 1(a).

   (b) Write an expression for the distribution of $\Delta\nu$ and $S\nu$ in the limit $N \gg 1$. Numerically test your result using simulations.

   (c) Show that the distribution of P^1 is independent of N in the limit $N \gg 1$.

   (d) **Advanced:** Compute analytically the distribution of P^1 and compare your results to the simulations.