SIR model

susceptible
infective
removed

$$\dot{I} = S \cdot I - \alpha I$$

Synaptic – inputs from other neurons

adaptatron

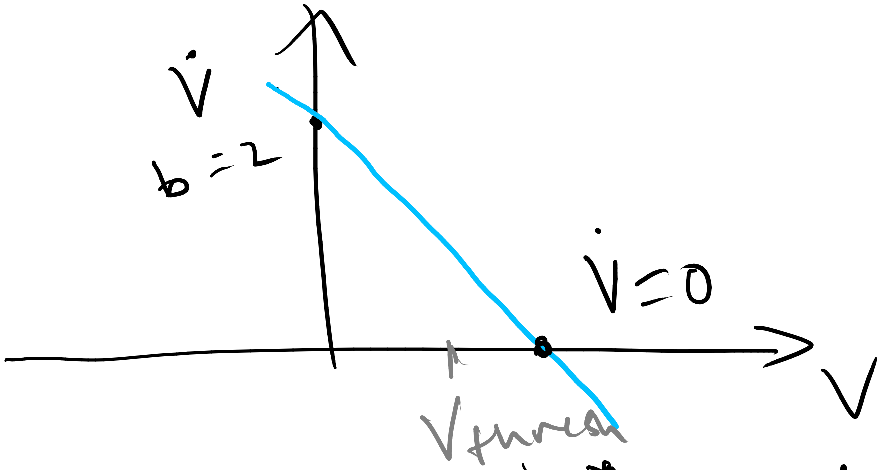
no pump

structure $V(t)$ point neuron

neuromodullutors

LIF model

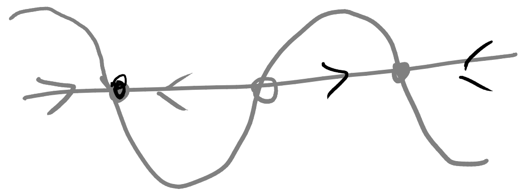
$\dot{V} = b - V$



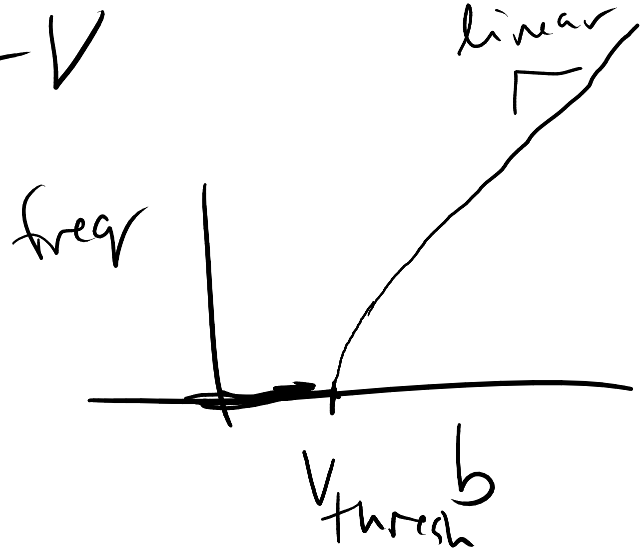
spiking

V^* fixed pt $\quad V^* = b$

$\dot{V} = f(x)$



Solve ODE $\dot{V} = b - V$



$$\int_0^T \frac{dV}{b-V} = \int_0^T dt$$

$$-\ln(b-V)\Big|_0^T = T$$

$$\ln(b - V(0)) - \ln(b - V(T)) = T$$

$b > V_{thresh}$

$$\begin{cases} V(T) = V_{thresh} \\ V(0) = 0 \quad \text{reset} \end{cases}$$

$$T_{spike} = \ln b - \ln(b - V_{thresh})$$
$$= -\ln\left(\frac{b - V_{thresh}}{b}\right) = -\ln\Big(1 - \underbrace{\frac{V_{thresh}}{b}}_{\text{small}}\Big)$$
$$\approx \frac{V_{thresh}}{b}$$

$$\text{freq} \approx b$$

$$\frac{d\vec{x}}{dt} = \overbrace{-\vec{x} + gJh(\vec{x})}^{f(\vec{x})}$$

$$0 = -\cancelto{0}{\vec{x}} + gJh(\cancelto{0}{\vec{x}})$$

$\vec{0} = \vec{x}$ is f.p.

$$f_i = -x_i + g\sum_k J_{ik}h(x_k)$$

$$(Df)\big|_{\vec{x}=0} = -I + gJ\cancelto{1}{h'(0)}$$

Jacobian matrix w/ entries $\frac{\partial f_i}{\partial x_j}$ $n \times n$

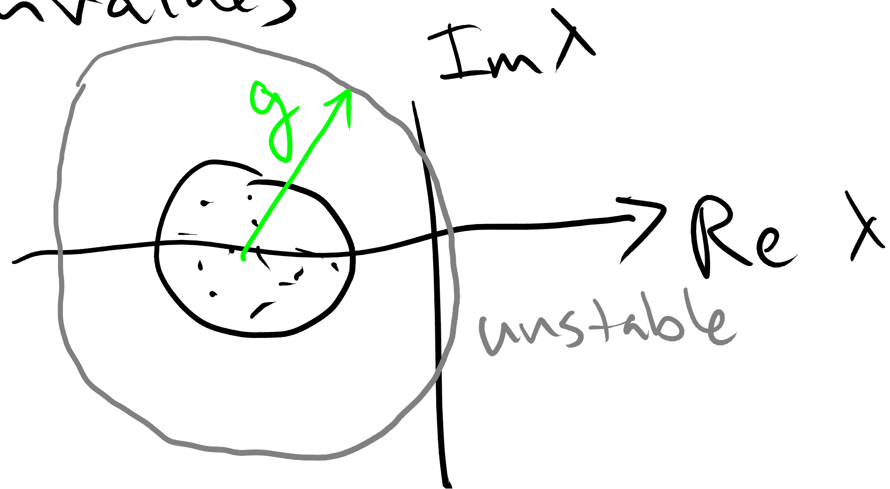
$$= -I + gJ$$

$$\frac{\partial f_i}{\partial x_j} = \dots + gJ_{ij}h'(x_j)$$

eigenvalues

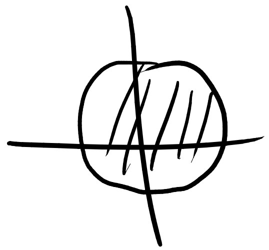
J random

~ circular law w/ radius



Circular law   Ginibre ; Tao, Vu

J random $n \times n$ matrix i.i.d entries
expected value $= 0$, var $= \frac{1}{\sqrt{n}}$

$\text{ESD} \xrightarrow{d}$  equal prob on unit circle

$n \to \infty$

$$A = -I + gJ$$

$$\lambda(A) = \lambda(J) \cdot g - 1$$

Hebbian Learning 1949

$$y(t) = \vec{w}(t-1)^T \, \vec{x}(t)$$

activity weights input

$$\vec{w}(t) = \vec{w}(t-1) + \Delta \vec{w}(t-1)$$

$$\Delta \vec{w}(t-1) = \underset{\text{step size}}{\gamma} \, y(t) \, \vec{x}(t)$$

Hebb's rule

Oja 1982

$$\Delta \vec{w}(t-1) = \gamma \, y(t) \left[\vec{x}(t) - \underbrace{y(t) \, \vec{w}(t-1)} \right]$$

stabilize ≃ approx of Gram-Schmidt

Hebb: $C = \mathbb{E}_t \left[\vec{x}(t) \vec{x}(t)^T \right]$

centered

$\vec{w}(t \to \infty) \propto$ leading eigenvec.

$$\frac{\vec{w}(t) - \vec{w}(t-1)}{\gamma} = y(t)\,\vec{x}(t)$$

$$= \left(\vec{w}^T(t-1)\,\vec{x}(t) \right) \underline{\underline{\vec{x}(t)}}$$

$$= \underline{\underline{\vec{x}(t)}}\,\vec{x}(t)^T\,\vec{w}(t-1)$$

$\Downarrow$

$$\frac{d\vec{w}}{dt} = C\,\vec{w} \longrightarrow \text{leading eig of } C$$

Perceptron

$\vec{x}_t$ input $\quad y_t \in \pm 1$ label

neuron computes

$$f(\vec{x}) = \vec{w}^T \vec{x} \longrightarrow \text{predict } \operatorname{sign}(f)$$

Perceptron rule: If $\operatorname{sign}(f(\vec{x}_t)) \neq y_t$

$$\vec{w}_{t+1} = \vec{w}_t + \gamma\, y_t \vec{x}_t$$

Thm Novikoff 1962, various 1960s

If data $\|\vec{x}_t\| \leq R$ and $\mu > 0$

margin $(\vec{w}^{*T} \vec{x}_i)\, y_i \geq \mu$

for some $\vec{w}^*$.

Then convergence in $\leq \left(\frac{R}{\mu}\right)^2$ steps.