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- ***Bayesian observer models***
- ***Cognitive illusions as Bayesian inference***

Problem set

1. **Probability distortion as Bayesian inference.** Suppose every trial you encounter a new slot machine that has a probability of p to deliver rewards. You task is to try the slot machine for $n = 5$ times and estimate its rewarding probability based on the observed rewarding times k. You know p follows a Beta distribution over trials: $p \sim Beta(3,4)$. If you are a Bayesian observer with a quadratic loss function (i.e., squared error), how will the mean of your estimated $\hat{p}$ vary as a function of p?
   (1) Write down your derivation and plot the results.
   (2) What if you are allowed to sample each slot machine for $n = 1,10, or\ 100$ times? Plot the results.
2. **It's about your own belief.** In **Problem 1**, if we still have $p \sim Beta(3,4)$ over trials but you incorrectly believe that $p \sim Beta(1,1)$ (i.e., a Uniform distribution), how will the mean of your estimated $\hat{p}$ vary as a function of p?
   (1) Compare the function under this incorrect prior belief with that under the correct prior belief. Plot the results.
   (2) Compute the resulting mean squared errors for each p and over different values of p, separately for the correct and incorrect prior beliefs. Compare between the two beliefs and plot the results.
3. **It also depends on how you define loss.** In **Problem 1**, if you use a 0-1 loss function (i.e., 0 if $\hat{p}$ matches p and 1 otherwise) instead of an L_2 loss function, how will the mean of your estimated $\hat{p}$ vary as a function of p? Plot your results.

- ***Bayesian observer models***

Abstract: Bayesian observer models refer to a range of computational models widely used in perception and cognition, where humans, animals or artificial agents are modeled as ideal observers whose inferences follow the Bayesian theorem and whose behaviors maximize expected gain. Given our CNeuro2022 schedule, I assume that you will have already had some experience with Bayesian observer

models and their applications in perception by the time of my lecture. This lecture will then focus on (1) the application of Bayesian observer models in higher-level cognition, and (2) practical issues in building and testing Bayesian observer models, including common mistakes. My goal is to provide a broader view of what Bayesian observer models are and how they may help us to deepen our understanding of perception and cognition.

- ***Cognitive illusions as Bayesian inference***

Abstract: It has long been known that many visual illusions can be explained as the consequence of Bayesian inference. More recently, Bayesian inference, in conjunction with the concept of efficient coding, has also been used to account for higher-level "cognitive illusions" (biases), such as the distortions of value and probability in decision making. In this advanced lecture on Bayesian observer models, I will discuss several such examples in recent literature. The basic idea is that the brain tries to use its limited cognitive resources efficiently and reduce the uncertainty of its internal representations, and in doing so leads to systematic cognitive biases.

Reading materials:

Anderson, J. R. (1991). The adaptive nature of categorization. *Psychological Review*, *98*(3), 409–429.

Polanía, R., Woodford, M., & Ruff, C. C. (2019). Efficient coding of subjective value. *Nature Neuroscience*, *22*(1), 134–142. https://doi.org/10.1038/s41593-018-0292-0

Bhui, R., & Gershman, S. J. (2018). Decision by sampling implements efficient coding of psychoeconomic functions. *Psychological Review*, *125*(6), 985–1001. https://doi.org/10.1037/rev0000123

Zhang, H., Ren, X., & Maloney, L. T. (2020). The bounded rationality of probability distortion. *Proceedings of the National Academy of Sciences*, *117*(36), 22024–22034. https://doi.org/10.1073/pnas.1922401117