A normative approach to neural computation

Dmitri "Mitya" Chklovskii

FLATIRON INSTITUTE
Division of Simons Foundation

NYU NEUROSCIENCE

Richard Feynman
1918-1988

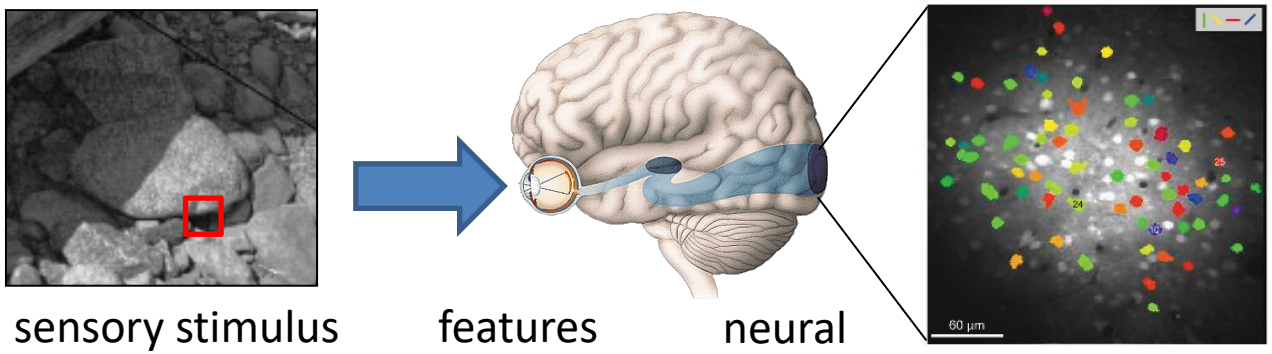
"In general we look for a new law by the following process. First we guess it. Then we compute the consequences of the guess to see what would be implied if this law that we guessed is right. Then we compute the result of the computation to nature, with experiment or experience, compare it directly with observation, to see if it works… if it disagrees with experiment it is wrong. That is all there is to it."

Energy function $\xrightarrow{\text{Optimization}}$ **Dynamics**

A normative approach to neural computation

- Reconstruction/decoding approach
  - Single-neuron PCA
  - Multiple neurons
- Similarity matching approach
  - Linear dimensionality reduction
  - Non-negative output
  - Two-compartment neurons

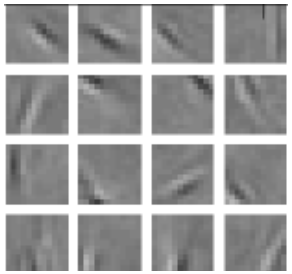
Reconstruction approach: linear decoding



sensory stimulus features neural activities

$$\boldsymbol{x} \approx \sum_k \boldsymbol{w}_k y_k$$

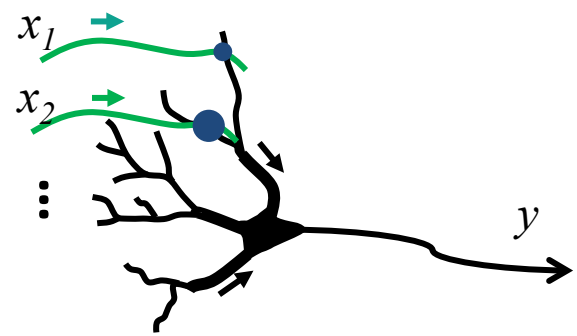
$\boldsymbol{w}_{1:n}$:



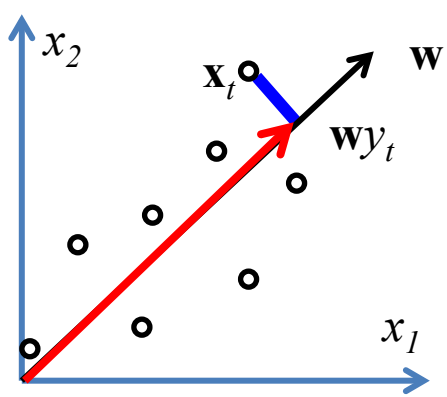
Abbott, Atick, Bialek, Olshausen & Field, …

Single neuron

$$\min_{\mathbf{w}, y_{1..T}} \sum_{t=1}^{T} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$$



Principal component analysis (PCA)

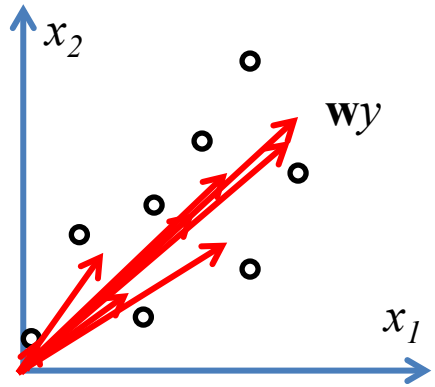


$$\boldsymbol{w} \sim 1^{st}\text{eig}\left(\sum_{t}^{T} \boldsymbol{x}_t \boldsymbol{x}_t^{\mathsf{T}} \right)$$

Online PCA

Oja, 1982; Yang, 1995

$$\min_{\mathbf{w}, y_{1..T}} \sum_{t=1}^{T} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$$



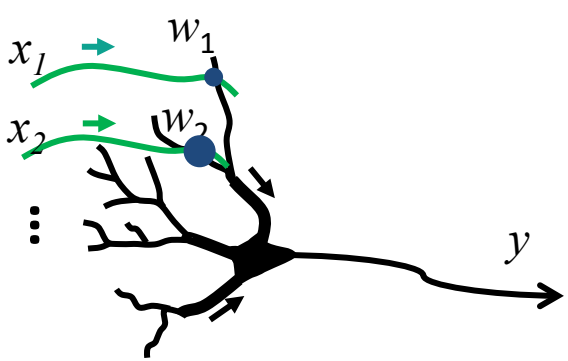
Phase I: neural activity

$$y_T = \arg\min_y \left\| \mathbf{x}_T - \mathbf{w}_{T-1} y \right\|_2^2$$

$$= \arg\min_y \left[-2\mathbf{w}_{T-1}^{\mathsf{T}} \mathbf{x}_T y + \left\| \mathbf{w}_{T-1} \right\|_2^2 y^2 \right]$$

$$= \arg\min_y \left[\frac{\mathbf{w}_{T-1}^{\mathsf{T}} \mathbf{x}_T}{\left\| \mathbf{w}_{T-1} \right\|_2^2} - y \right]^2 \left\| \mathbf{w}_{T-1} \right\|_2^2$$

$$\boxed{y_T = \mathbf{w}_{T-1}^{\mathsf{T}} \mathbf{x}_T \Big/ \left\| \mathbf{w}_{T-1} \right\|_2^2}$$



Output = weighted input

Online PCA learning rule

Oja, 1982; Yang, 1995

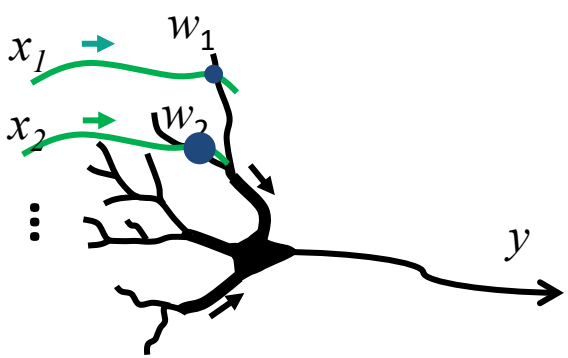
$$\min_{\mathbf{w}, y_{1..T}} \sum_{t=1}^{T} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$$

Phase II: synaptic weight

$$\mathbf{w}_T = \arg\min_{\mathbf{w}} \sum_{t=1}^{T} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$$

$$= \arg\min_{\mathbf{w}} \sum_{t=1}^{T} \left[-\mathbf{w}^{\mathsf{T}} \mathbf{x}_t y_t + \left\| \mathbf{w} \right\|_2^2 y_t^2 \right]$$

$$= \arg\min_{\mathbf{w}} \left\| \frac{\sum_{t=1}^{T} \mathbf{x}_t y_t}{\sum_{t=1}^{T} y_t^2} - \mathbf{w} \right\|_2^2 = \frac{\sum_{t=1}^{T} \mathbf{x}_t y_t}{\sum_{t=1}^{T} y_t^2}$$



$$\boxed{\mathbf{w}_T = \mathbf{w}_{T-1} + y_T \left(\mathbf{x}_T - \mathbf{w}_{T-1} y_T \right) \Big/ \sum_{t=1}^{T} y_t^2}$$

Hebbian synaptic learning rule

Online PCA: single component

Optimization problem

Offline: $\min_{\mathbf{w}} \sum_{t=1}^{T} \min_{y_t} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$

Online: $y_T = \arg\min_{y} \left\| \mathbf{x}_T - \mathbf{w}_{T-1} y \right\|_2^2 ;$ $\mathbf{w}_T = \arg\min_{\mathbf{w}} \sum_{t=1}^{T} \left\| \mathbf{x}_t - \mathbf{w} y_t \right\|_2^2$

Online algorithm

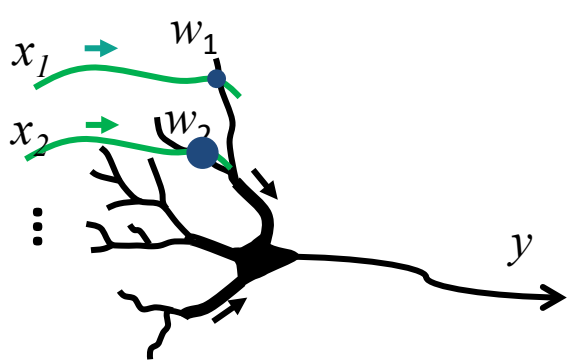
neural activity: $y_T = \mathbf{w}_{T-1}^{\mathsf{T}} \mathbf{x}_T \Big/ \left\| \mathbf{w}_{T-1} \right\|_2^2$

(Oja, 1982; Yang, 1995)

synaptic weight: $\mathbf{w}_T = \mathbf{w}_{T-1} + y_T \left(\mathbf{x}_T - \mathbf{w}_{T-1} y_T \right) \Big/ \sum_{t=1}^{T} y_t^2$

Local Hebbian learning rule

Neuron model

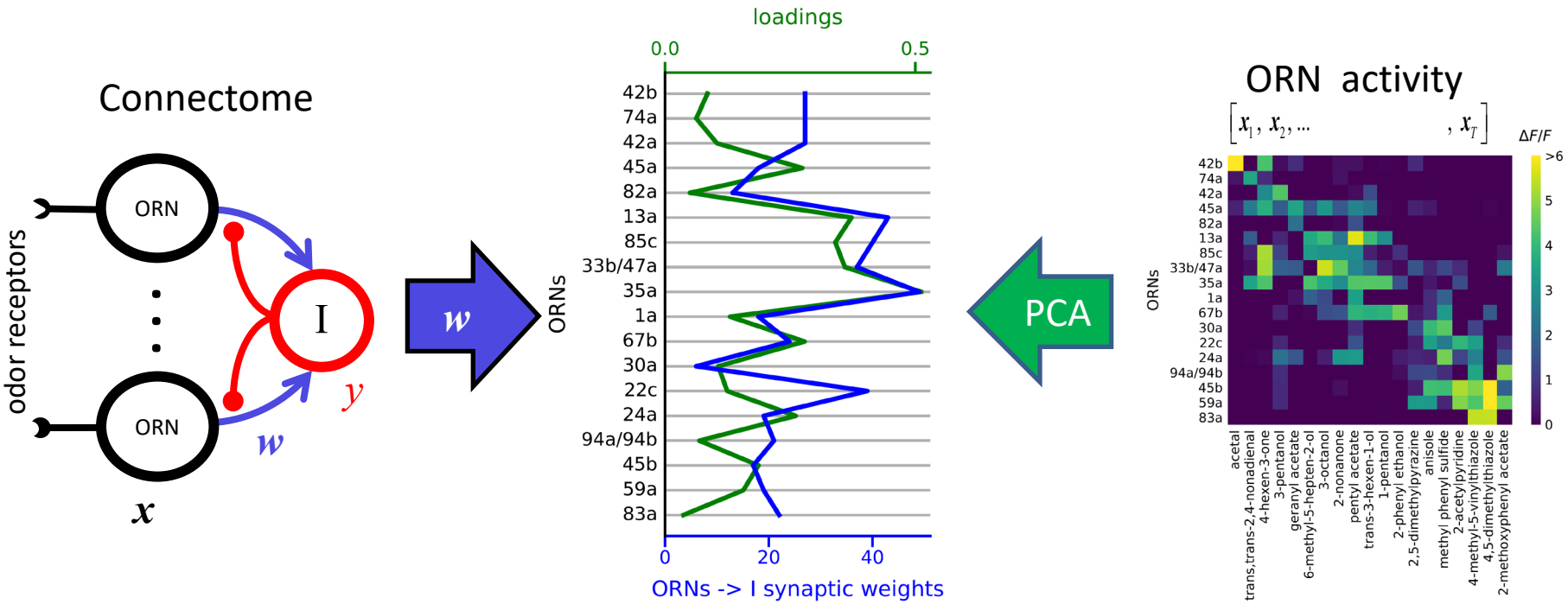


Prediction:

synaptic weight vector ~ top PC of neural activity

$$\boldsymbol{w} \sim 1^{st}\mathrm{eig}\left(\sum_{t}^{T} \boldsymbol{x}_t \boldsymbol{x}_t^{\mathsf{T}}\right)$$

Experimental test in fly larva olfactory system



Berck et al, 2016

Chapochnikov, Pehlevan, Chklovskii, submitted

Si et al, 2019

Dependence of synaptic plasticity on age and activity

Theory

$$\mathbf{w}_T = \mathbf{w}_{T-1} + y_T\left(\mathbf{x}_T - \mathbf{w}_{T-1}y_T\right)\Big/\sum_{t=1}^{T} y_t^2$$

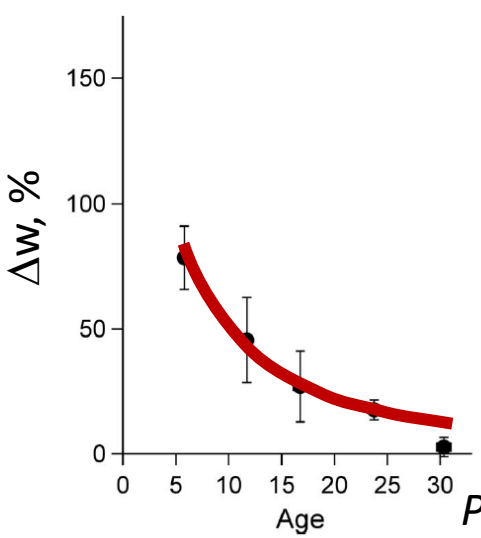
Nonstationary input statistics -> discounting of past errors or forgetting:

$$\mathbf{w}_T = \arg\min_{\mathbf{w}}(1-\alpha)\sum_{t=1}^{T}\alpha^{T-t}\left\|\mathbf{x}_t - \mathbf{w}y_t\right\|_2^2,$$

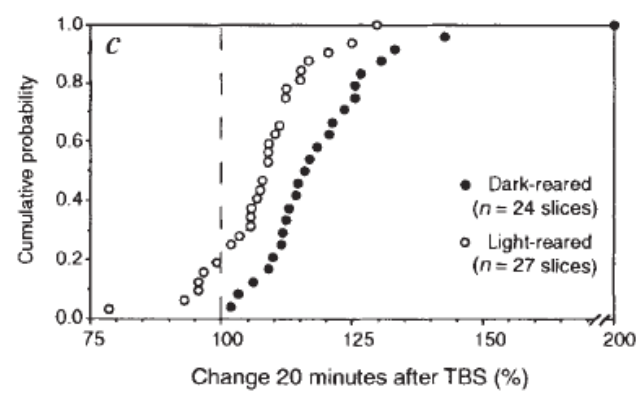
$$\mathbf{w}_T = \alpha\mathbf{w}_{T-1} + y_t\left(\mathbf{x}_t - \mathbf{w}y_t\right)\Big/\sum_{t=1}^{T}\alpha^{T-t}y_t^2$$

$\alpha = e^{-1/\tau_w} < 1, \quad \tau_w$ - autocorrelation time of $\mathbf{w}$

Experiment (Long-Term Potentiation)

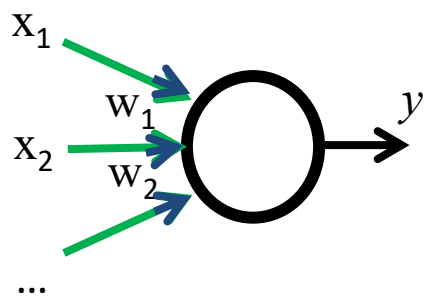


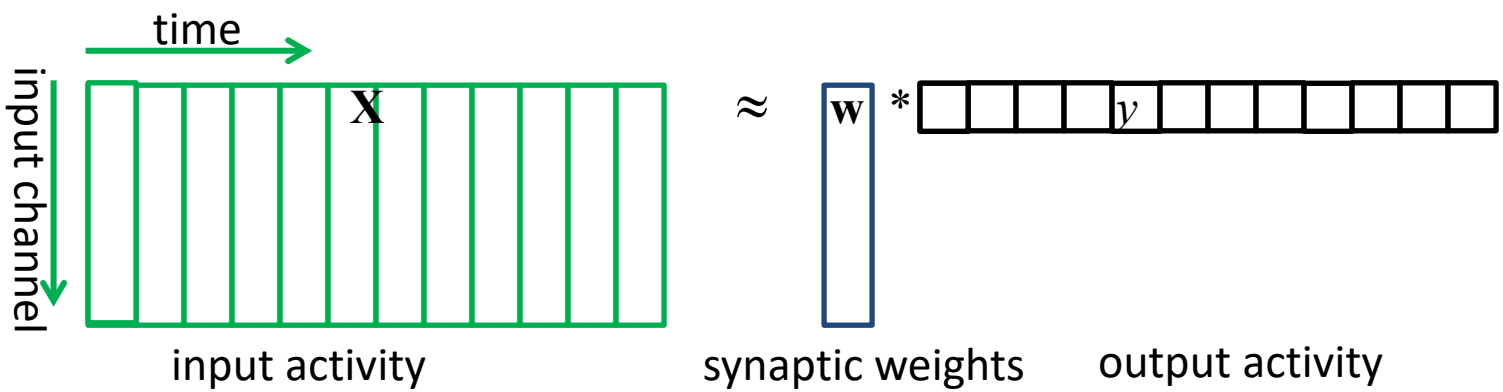
Poo & Isaacson 2007



Kirkwood, Lee, Bear 1995

A neuron performs online matrix factorization





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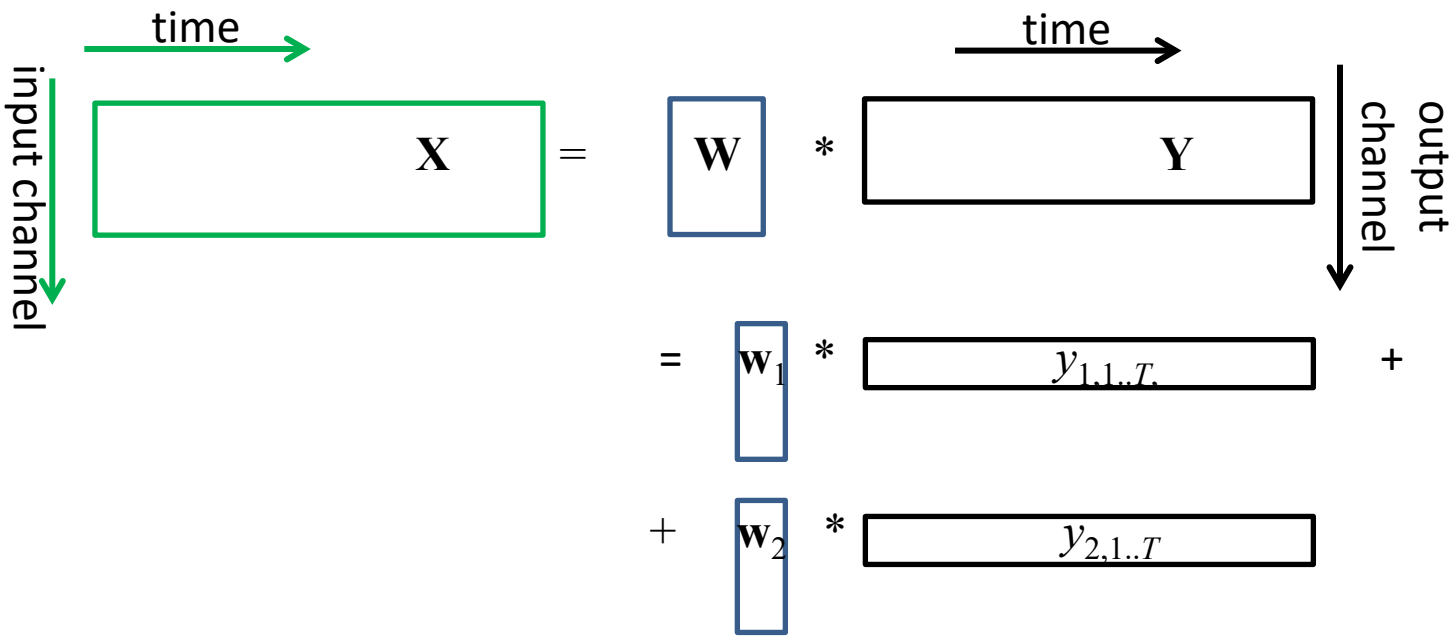
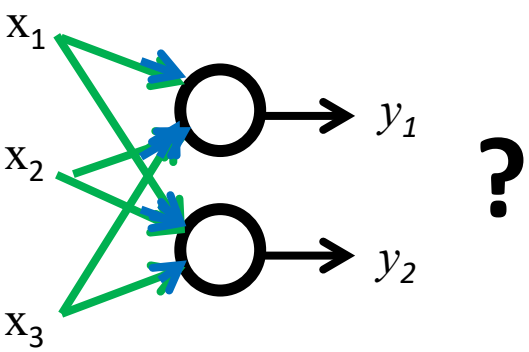
Multiple principal components?





multiple neurons

Online dimensionality reduction

Optimization problem

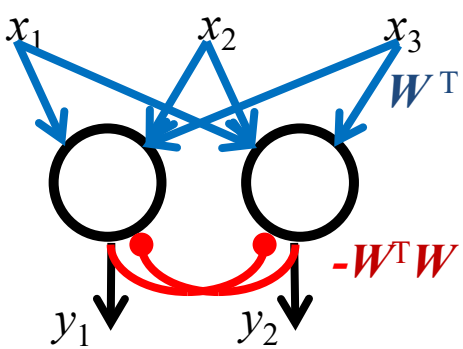
Offline: $$\min_{\mathbf{W}} \sum_{t=1}^{T} \min_{\boldsymbol{y}_t} \left\| \mathbf{x}_t - \mathbf{W}\boldsymbol{y}_t \right\|_2^2$$

Online algorithm

neural activity: Iterate $$\boldsymbol{y}_T \leftarrow \boldsymbol{W}_{T-1}^{\mathsf{T}} \boldsymbol{x}_T - \boldsymbol{W}_{T-1}^{\mathsf{T}} \boldsymbol{W}_{T-1} \boldsymbol{y}_T$$

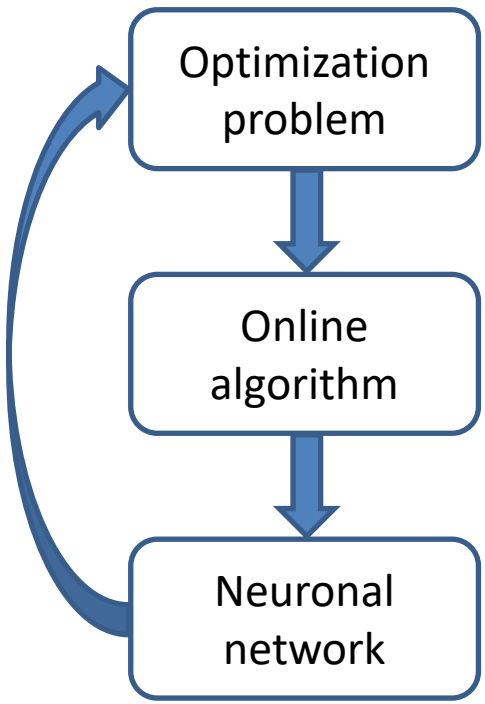
synaptic weight: $$W_{j,i,T} \leftarrow W_{j,i,T-1} + \eta y_{i,T} \left[x_{j,T} - \sum_k W_{j,k,T-1} y_{k,T} \right]$$

Neuronal network



Online dimensionality reduction

No objective or function	Biologically implausible nonlocal learning rules
Foldiak (1990), Oja (1992), Zylberberg et al (2011)	*Bell & Sejnowski (1997), Falconbridge et al (2005), Gerhard et al (2009), Olshausen & Field (1996)*



Offline: $$\min_{\mathbf{W}} \sum_{t=1}^{T} \min_{\boldsymbol{y}_t} \left\| \mathbf{x}_t - \mathbf{W}\boldsymbol{y}_t \right\|_2^2$$

neural activity: Iterate $$\boldsymbol{y}_T \leftarrow \boldsymbol{W}_{T-1}^{\mathsf{T}} \boldsymbol{x}_T - \boldsymbol{W}_{T-1}^{\mathsf{T}} \boldsymbol{W}_{T-1} \boldsymbol{y}_T$$

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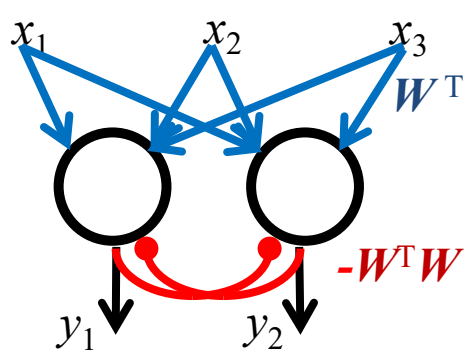

Alternative two-layer reciprocal circuit: Koulakov and Rinberg, 2011; Druckmann, Hu, Chklovskii, 2012

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  - Multiple neurons -> difficulties
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Acknowledgements

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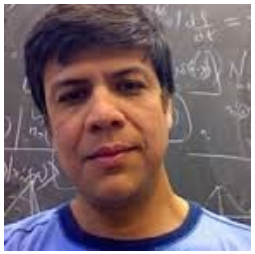

Anirvan Sengupta

David Lipshutz

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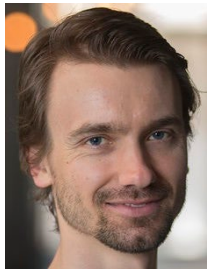
-> Google



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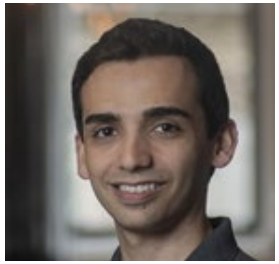
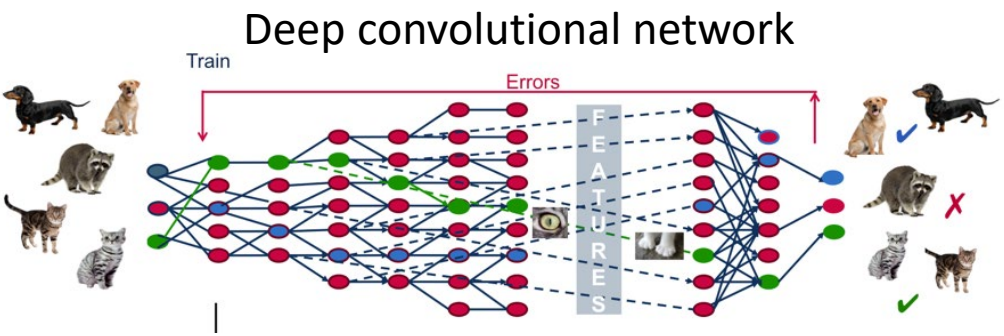
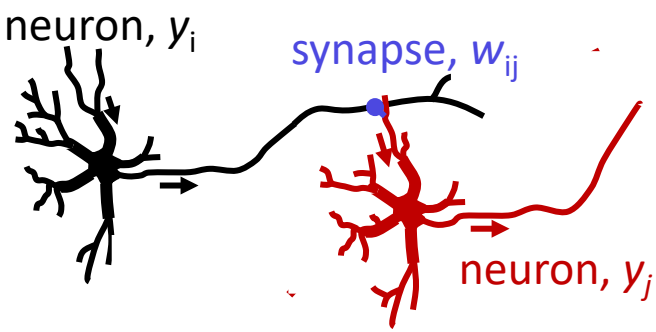


Siavash Golkar

Neural networks: artificial and natural





- Supervised, trained on huge labeled datasets
- Nonlocal learning rules, backpropagation
- …
- Poorly understood

- Mostly unsupervised, learning from few labeled examples
- Local learning rules: $w_{ij} = f(y_i, y_j)$
- …
- Poorly understood